EXAMPLES OF SECTIONS 3.3, 3.5

Question 1. Find the determinant of

$$A = \begin{bmatrix} 3 & 5 & 6 \\ 2 & 4 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

Question 2. Solve the system

$$A\vec{x} = \vec{b},$$

where A is the matrix of question 1 and

$$\vec{b} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix},$$

by using the Cramer's rule.

SOLUTIONS.

1. We have

$$A_{11} = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix} \Rightarrow \det(A_{11}) = 4 \cdot 5 - 3 \cdot 3 = 11,$$

$$A_{12} = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix} \Rightarrow \det(A_{12}) = 2 \cdot 5 - 3 \cdot 2 = 4,$$

$$A_{13} = \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix} \Rightarrow \det(A_{13}) = 2 \cdot 3 - 2 \cdot 4 = -2.$$

Hence,

$$\det(A) = 3\det(A_{11}) - 5\det(A_{12}) + 6\det(A_{13}) = 33 - 20 - 12 = 1.$$

2. Replace the first column of A with $\vec{b}$ to get:

$$A_1(\vec{b}) = \begin{bmatrix} 0 & 5 & 6 \\ 2 & 4 & 3 \\ 0 & 3 & 5 \end{bmatrix}.$$

Analogously, replacing the second and third column produces

$$A_2(\vec{b}) = \begin{bmatrix} 3 & 0 & 6 \\ 2 & 2 & 3 \\ 2 & 0 & 5 \end{bmatrix},$$

and

$$A_3(\vec{b}) = \begin{bmatrix} 3 & 5 & 0 \\ 2 & 4 & 2 \\ 2 & 3 & 0 \end{bmatrix}$$

Then

$$\det(A_1(\vec{b})) = 0 \cdot \det \begin{bmatrix} 4 & 2 \\ 3 & 0 \end{bmatrix} - 2 \cdot \det \begin{bmatrix} 5 & 6 \\ 3 & 5 \end{bmatrix} + 0 \cdot \det \begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix} = -14.$$

Analogously,

$$\det(A_2(\vec{b})) = -0 \cdot \det \begin{bmatrix} 2 & 3 \\ 2 & 50 \end{bmatrix} + 2 \cdot \det \begin{bmatrix} 3 & 6 \\ 2 & 5 \end{bmatrix} - 0 \cdot \det \begin{bmatrix} 3 & 6 \\ 2 & 3 \end{bmatrix} = 6,$$

$$\det(A_3(\vec{b})) = 0 \cdot \det \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix} - 2 \cdot \det \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix} + 0 \cdot \det \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} = 2.$$

We finally obtain

$$x_1 = \frac{\det(A_1(\vec{b}))}{\det(A)} = \frac{-14}{1} = -14,$$

$$x_2 = \frac{\det(A_2(\vec{b}))}{\det(A)} = \frac{6}{1} = 6,$$

$$x_3 = \frac{\det(A_3(\vec{b}))}{\det(A)} = \frac{2}{1} = 2.$$